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Technical Note

Inlet temperature influence on the departure from Darcy flow of a fluid with variable viscosity

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1. Introduction

In this note, the effect of inlet temperature on the departure from linear Darcy flow to quadratic (form-drag dominant) flow regime is studied. By using the results of numerical simulations for poly-alpha-olefin, a synthetic oil, and the M-HDD model [1] the departure from linear to quadratic flow regime (transition) is shown to happen at an earlier global velocity than for the constant viscosity counterpart, when the channel is heated. By performing the simulations within the same parameter (velocity and heat flux) range, the effect of inlet temperature on the shift in transition is revealed. This aspect is of practical importance because most fluids have viscosity non-linearly dependent on temperature.

The transition criterion used in this study is based on the ratio of global form-drag and viscous-drag forces along a porous channel with uniform cross-section, given by

$$\lambda = \frac{D_{C_0}}{D_{\mu_0}} = \left(\frac{\rho C_0 K_0}{\mu_0}\right) U, \tag{1}$$

where K_0 and C_0 are the permeability and form coefficient of the porous medium obtained from isothermal experiments, U is the cross-section averaged Darcy (or seepage) fluid speed, $D_{C_0} = \rho C_0 U^2$ represents the global form-drag and $D_{\mu_0} = \mu_0 U/K_0$ represents the global viscous-drag (with viscosity evaluated at the fluid temperature, i.e., $\mu_0 = \mu(T_0)$) acting within the porous medium.

Observe that to interpret the results of Eq. (1) correctly, the HDD model

$$\left.\frac{\Delta P}{L}\right|_0 = \frac{\mu(T_{\text{in}})}{K_0} U + \rho C_0 U^2 = D_{\mu_0} + D_{C_0}, \tag{2}$$

where $\frac{\Delta P}{L}|_0$ refers to the pressure-drop across the channel for isothermal flows, should be valid.

2. The role of temperature-dependent viscosity (the modified-HDD model)

Numerical simulations considering convection of a fluid with temperature-dependent viscosity through a uniformly heated, parallel-plates porous channel, and including the form-drag effects, were presented recently by Narasimhan and Lage [1]. In this work, the authors showed the limitations of the global HDD model, Eq. (2), in accurately predicting the pressure-drop along the channel, suggesting a modification to account for the temperature-dependent viscous effects. They also showed that the global HDD model, Eq. (2), is inappropriate because it neglects indirect effects of temperature-dependent viscosity on the form-drag term of the model, a term originally believed to be viscosity independent.

A new global model

$$\frac{\Delta P}{L} = \zeta_\mu \left(\frac{\mu_0}{K_0}\right) U + \zeta_C (\rho C_0) U^2$$
$$= \zeta_\mu D_{\mu_0} + \zeta_C D_{C_0} = D_\mu + D_C \tag{3}$$

that accounts for the effects of temperature-dependent viscosity in both drag terms of the original HDD model, was proposed.

Notice that this model *retains* the same form, i.e., velocity dependency, of Eq. (2). The velocity independent

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coefficients ζ_μ and ζ_C represent, respectively, the correction for the global viscous-drag and form-drag terms due to the local effect of temperature on viscosity, which affect directly the first-order velocity term and the fluid velocity profile (via viscosity), which in turn, affects indirectly the second-order velocity term. Obviously, for a no heating configuration (uniform viscosity), $\zeta_\mu = \zeta_C = 1$ and Eq. (3) becomes identical to Eq. (2).

Preliminary predictive empirical relations for correcting the viscous- and form-drag terms, complementing this new algebraic (global) model, were obtained [1] as functions of the surface heat flux,

$$\zeta_\mu = \left[1 - \left(\frac{Q''}{1+Q''}\right)^{0.325}\right]\left(\frac{1}{1+Q''}\right)^{18.2}, \qquad (4a)$$
$$\zeta_C = 2 + (Q'')^{0.11} - \zeta_\mu^{-0.06}$$

with Q'' given by

$$Q'' = \frac{q''}{\left(\frac{k_e}{K_0 C_0}\right)\mu_{\text{in}}}\left|\frac{\mathrm{d}\mu}{\mathrm{d}T}\right|_{T_{\text{in}}}. \qquad (4b)$$

The physics of the problem is detailed in [1] and is not repeated here for brevity.

Finally, defining

$$\Phi = \frac{(\Delta P/L)}{(\Delta P/L)|_0} = \frac{D_\mu}{(D_{\mu_0} + D_{C_0})} + \frac{D_C}{(D_{\mu_0} + D_{C_0})} = \Phi_\mu + \Phi_C \qquad (5)$$

as a non-dimensional pressure-drop clearly highlights the viscosity variation effect, as it compares the pressure-drop got by considering viscosity variation (Eq. (3)) to that for uniform viscosity (Eq. (2)). Observe for the case of a fluid flowing with uniform viscosity then $\zeta_\mu = \zeta_C = 1$ and, from Eq. (3), $\Delta P/L$ equals $(\Delta P/L)_0$, thus yielding $\Phi = 1$. In addition, notice from Eq. (1), for a given porous configuration (i.e., for a chosen set of ρ, μ_0, K_0 and C_0), λ is a function only of U. These variables (Φ_μ, Φ_C and λ) are utilized in the plots of Figs. 1–3.

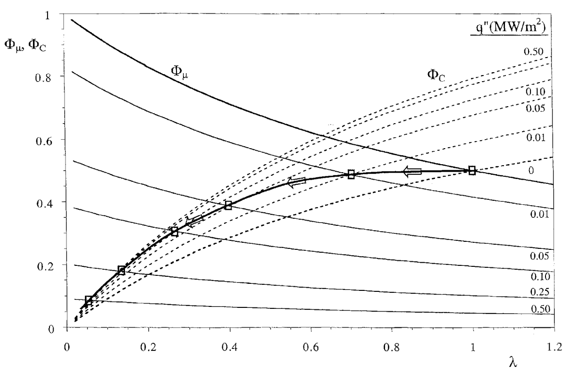

Fig. 1. Shifts in the transition point for fluid with $T_{\text{in}} = 7$ °C.

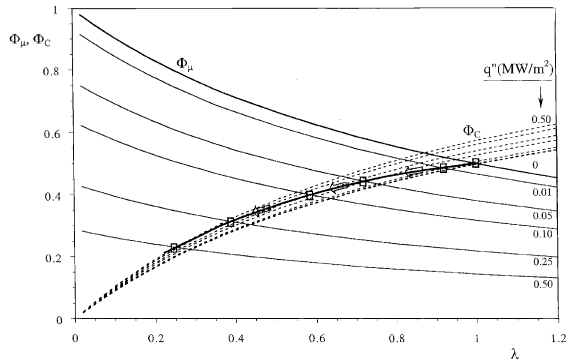

Fig. 2. Shifts in the transition point for fluid with $T_{\text{in}} = 21$ °C.

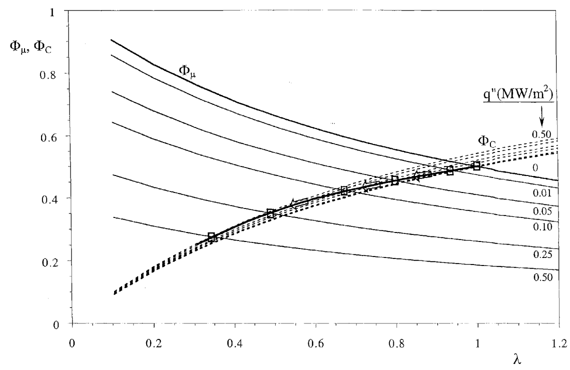

Fig. 3. Shifts in the transition point for fluid with $T_{\text{in}} = 32$ °C.

With these, we are now equipped to study the problem of departure from Darcy flow in a configuration with temperature-dependent viscosity effects.

3. Simulation details

To get the necessary pressure-drop versus global velocity results, numerical simulations were performed for an isoflux parallel plate porous channel ($K_0 = 4.1 \times 10^{-10}$ m^2 and $C_0 = 1.2 \times 10^5$ m^{-1}), using the differential forms of the HDD model and the energy equation. Details of the flow configuration and of the numerical procedure (tested for grid independence and numerical accuracy) can be found in [1].

The dynamic viscosity of PAOs, the fluid considered in our study, can be modeled [2] as

$$\mu(T) = 0.1628\, T^{-1.0868} \ (\text{N s m}^{-2}) \qquad (6)$$

valid for 5 °C $\leqslant T \leqslant$ 170 °C. Observe the inverse temperature dependence of PAO viscosity results in large temperature gradation at low temperatures. For instance $\mu = 19.64 \times 10^{-3}$, 5.95×10^{-3} and 3.77×10^{-3} for PAO temperatures 7, 21 and 32 °C, respectively. Within the same temperature range, the variations of density, specific heat and thermal conductivity of PAO are negligible (see [3]).

4. Inlet temperature influence on the transition

Fig. 1 displays the viscosity effects on the transition with increasing heat flux, for an inlet temperature of 7 °C. The continuous curves represent the non-dimensional global viscous-drag values (Φ_μ) and the dashed lines represent the corresponding global form-drag values (Φ_C). For any velocity (λ), the sum of the corresponding drag values will give the total pressure-drop experienced by the flow across the channel. For no heating (constant viscosity) case, $q'' = 0$, this pressure-drop would equal the result of the HDD model, Eq. (2), leading to $\Phi = 1$ (Eq. (5)), and when heating and $\mu = \mu(T)$, it would equal the result of the M-HDD model, Eq. (3).

For no heating ($q'' = 0$), with increasing velocity, the global viscous-drag decreases with a corresponding increase in strength of the global form-drag – as expected for the constant viscosity case. This portrays the gaining dominance of the non-linear, form-drag effect as the velocity increases. The curves cross for $\lambda = 1$ (λ_T), marked in Fig. 1 with a square, highlighting the equivalence in strength of the drags. Beyond this point (i.e., for all higher velocities) the global form-drag predominates.

For the heat flux $q'' = 0.01$ MW m^{-2}, the pair of continuous and dashed lines meet at an *earlier* point (in terms of λ) than for the no-heating constant viscosity case (also marked with a square, Fig. 1). The shift is caused not only because of the direct influence of viscosity change (decrease, in our case) with temperature but also of the change in the global form-drag caused by variation in the local velocity profile, an indirect influence of the changing viscosity (see [1]).

Notice also from Fig. 1, for $q'' = 0.01$ MW m^{-2}, the transition point shift almost *horizontally* to the left. This suggests that the reduction in the global viscous-drag caused by the local viscosity reduction everywhere in the channel is *equally* offset by a corresponding global form-drag increase.

Figs. 2 and 3 are for inlet temperature 21 and 32 °C, respectively. Going from Figs. 1–3, we can observe that the shift of the transition point to an earlier velocity when q'' goes from 0 to 0.01 MW m^{-2}, is less pronounced, as the inlet temperature increases. Further, as the inlet temperature increases, it takes a larger amount of heating to observe the same degree of shift in the transition point. Notice in Figs. 2 and 3 (for 21 and 32 °C, respectively), how the curve depicting the transition point shift has almost identical curvature, but different from the one in Fig. 1 (for 7 °C). This is a direct consequence of the functional dependence of viscosity on temperature (see Eq. (6)).

In general, for a particular heat flux crossing the channel wall, for a chosen velocity (λ, in our figures) the fluid can be in linear (viscous-drag dominant) or non-linear (form-drag dominant) regime based on the fluid inlet temperature. For example, at $\lambda \sim 0.8$, Fig. 1 (for 7 °C) tells that the fluid (PAO) will be in a non-linear (form-drag dominant) regime for $q'' = 0.01$ MW m^{-2} whereas both Figs. 2 and 3 (for 21 and 32 °C, respectively) still predict a linear regime.

By following the block arrow marks in Figs. 1–3, it is evident that, in general, as the heat flux increases, the *transition*, for fluids with viscosity decreasing with temperature, occurs at lesser and lesser velocity values. As the local viscosity decreases further, for sufficiently higher heat fluxes, the effect of the global viscous-drag would become so negligible that the flow practically is *always* form-drag dominant. This conclusion is particularly useful from an engineering standpoint.

5. Predicting transition: the inlet temperature effect

Next, we focus on how to predict the transition point for fluid flow with temperature-dependent viscosity. From the definition Eq. (1), it is clear that λ is the ratio of the global drag terms of the global HDD model, Eq. (2). For a uniform viscosity flow (use of HDD model is valid), when these two drag values are comparable, we get the transition point $\lambda_T \sim 1$, beyond which, the flow becomes form-drag dominant. However, when this model is superceded by the more general M-HDD model, Eq. (3), which accounts for temperature-dependent viscosity effects, it follows that the transition point happens only when the global drag terms of Eq. (3) are comparable. That is, we must use the balance of the two drag terms on the RHS of Eq. (3) (instead of Eq. (2)). Doing so

$$\zeta_\mu D_{\mu_0} \sim \zeta_C D_{C_0} \tag{7}$$

would result in

$$\lambda_T|_{\mu(T)} = \frac{\zeta_\mu}{\zeta_C}. \tag{8}$$

Eq. (8) gives the $\lambda_T|_{\mu(T)}$, beyond which, the flow becomes form-drag dominant for flows with temperature-dependent viscosity effects. Since $\zeta_\mu < 1$ and $\zeta_C > 1$ always (see Eqs. (4a) and (4b), the transition point for temperature-dependent viscosity flows, as predicted by Eq. (8), is always *less* than that for the constant viscosity case (i.e., $\lambda_T = 1$). This conclusion is physically consistent and well supported by Figs. 1–3. Notice also that for uniform viscosity (i.e., for $q'' = 0$), ζ_μ and ζ_C are identically equal to unity. This makes the prediction of Eq. (8) consistent with the previous result (i.e., $\lambda_T|_{\mu(T)} = \lambda_T = 1$) got by using the equivalence of drags in the HDD model, Eq. (2).

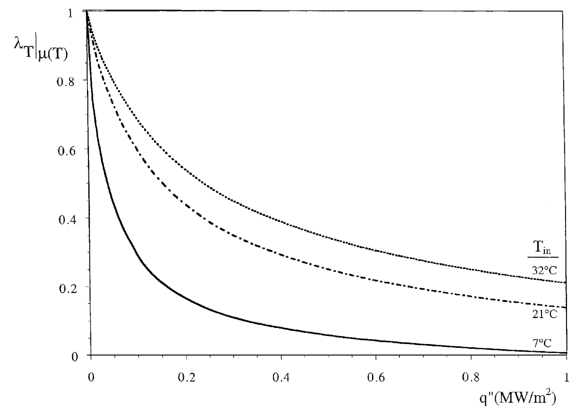

Fig. 4. Transition point for temperature-dependent viscosity: $\lambda_T|_{\mu(T)}$ versus heat flux.

Fig. 4 depicts the variation of $\lambda_T|_{\mu(T)}$ with heat flux, for different inlet temperatures. The curves show how for increasing heat flux, the transition point is shifted (from 1, for constant viscosity – no heating – case) to values less than 1, when temperature-dependent viscosity effects are included. Keep in mind, irrespective of the inlet temperature of the flow, if we assume viscosity is constant, $\lambda_T|_{\mu(T)}$ is *always* equal to unity, immaterial of the amount of heating. For a particular heat flux, it is apparent that the flow with $T_{\rm in} = 7$ °C becomes form-drag dominant earlier than for other higher inlet temperatures.

Observe also in Fig. 4, that the flow with $T_{\rm in} = 7$ °C asymptotically reaches zero for $q'' \sim 1.0$ MW m^{-2}. This means temperature-dependent viscosity effects on the viscous-drag term makes it practically equal to zero (i.e., in Eq. (8), $\lambda_T|_{\mu(T)} \to 0$ as the numerator $\zeta_\mu \to 0$). This makes the flow purely form-drag dependent (notice the use of the word "dependent" as against the original "dominant") for all higher heat fluxes.

Another interesting point to keep in mind is, though the temperature-dependent viscosity effect cannot affect the global viscous-drag term anymore, it is not restricted in influencing the global form-drag. The global form-drag can still be influenced by the velocity profile variation caused by the local viscosity variation (i.e., ζ_C can still be a non-zero positive number). This effect, the main claim of the M-HDD model, is fundamental to the physics of flow through porous media by fluids with temperature-dependent viscosity (see [1]).

6. Conclusion

Recent studies have shown that the global form-drag, though viscosity independent, is affected by changes in fluid viscosity with temperature invalidating the use of the HDD model for predicting transition. The effect of inlet temperature on the departure from linear Darcy flow to quadratic (form-drag dominant) flow regime is studied using the results of numerical simulations for poly-alpha-olefin, and the recently proposed M-HDD model that incorporates temperature-dependent viscosity effects in the prediction of global pressure-drop across a heated porous medium channel. The departure from linear Darcy flow to non-linear (form-drag dominant) flow regime is shown to happen at an earlier global velocity than for the constant viscosity counterpart. As the inlet temperature increases, the shift of the transition point to an earlier velocity happens to a lesser degree. Further, as the inlet temperature increases, it takes a large amount of heating to observe the same degree of shift in the transition point. Finally, a way to estimate the transition point using the coefficients of the M-HDD model, Eq. (8), is also shown.

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